BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: MCA
BRANCH: All
TIME: 3.00 HOURS

SEMESTER: I
SESSION: 2012 - 13(MO/12)
FULL MARKS: 60

SUBJECT: MMA 1115 DISCRETE MATHEMATICS

INSTRUCTIONS:
1. This question paper contains 7 questions each of12 marks and total 84 marks.
2. Candidates may attempt any 5 questions maximum of 60 marks.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph Paper etc. to be supplied to the candidates in the examination hall.

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1. a) Write the following statements in symbolic form: [4]
(i) If Avinash is not in a good mood or he is not busy, then he will go to Kharagpur.
(ii) If Sayantan knows object-oriented programming and oracle, then he will get a job.

b) Construct a truth table and investigate its tautology of the following compound proposition:
$(p \wedge q) \vee (p \vee r)$ [8]

2. a) Find a non-empty set and a relation on the set that satisfy each of the following combinations of properties. Simultaneously, draw a digraph of each relation: [6]
i) Reflexive and transitive but not symmetric.
ii) Reflexive and transitive but not antisymmetric.

b) Define Transitive Closure. Let $A = \{1, 2, 3, 4\}$ and $R = \{(1,2), (2,3), (3,4), (2,1)\}$. Find the transitive closure of R by matrix representation. [6]

3. a) Using principle of mathematical induction, show that
$$a + ar + ar^2 + ... + ar^{n-1} = \frac{a(1-r^n)}{1-r}, \quad r \neq 0.$$ [6]

b) Let $N = n^2 + n + 41$. Show that there are some values of n for which N is a prime number, and others for which it is not. It follows that there is no inductive step which would show that $n^2 + n + 41$ is a prime number for all possible n. [6]

4. a) Let $f : A \to B$ and $g : B \to C$ be two functions. Show that
i) if both f and g are injective, then $g \circ f$ is injective [6]
ii) if both f and g are surjective, then $g \circ f$ is surjective.

b) If $f : A \to B$ and $g : B \to C$ and $h : C \to D$, then show that
$$h \circ (g \circ f) = (h \circ g) \circ f$$ [6]

5. Write short notes of the following:
(i) Labelled Trees (ii) Tree Searching (iii) Spanning Trees [12]

6. a) If $(G_1, *_1)$ and $(G_2, *_2)$ are groups, then show that $G = G_1 \times G_2$ i.e., $(G, *)$ is a group with binary operation '$*$' defined by
$(a_1, b_1) * (a_2, b_2) = (a_1 *_1 a_2, b_1 *_2 b_2)$ [8]

b) Consider the set Q of rational numbers, and let '$*$' be the operation on Q defined by $a * b = a + b - ab$. Show that $(Q, *)$ is a semigroup. [4]

7. i) Let $(G,*)$ be a group and let $a, b \in G$. Then, show that
(i) $(a^{-1})^{-1} = a$ (ii) $(a * b)^{-1} = b^{-1} * a^{-1}$ [6]

ii) If $(G,*)$ is a group and if $a, b \in G$, then prove that [6]
(i) the equation $a * x = b$ has a unique solution $x = a^{-1} * b \in G$
(ii) the equation $y * a = b$ has a unique solution $y = b * a^{-1} \in G$.

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----- 10.11.2012 E -----