BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCH! (END SEMESTER EXAMINATION)

CLASS: MCA BRANCH: All TIME: 3.00 HOURS SEMESTER: I SESSION: 2012 - 13(MO/12) **FULL MARKS: 60**

SUBJECT: MMA 1115 DISCRETE MATHEMATICS

INSTRUCTIONS:

- 1. This question paper contains 7 questions each of 12 marks and total 84 marks.
- Candidates may attempt any 5 questions maximum of 60 marks.
- 3. The missing data, if any, may be assumed suitably.
- Before attempting the question paper, be sure that you have got the correct question paper.
- Tables/Data hand book/Graph Paper etc. to be supplied to the candidates in the examination hall.



Write the following statements in symbolic form:

- (i) If Avinash is not in a good mood or he is not busy, then he will go to Kharagpur.
- (ii) If Sayantan knows object-oriented programming and oracle, then he will get a job.
- Construct a truth table and investigate its tautology of the following compound proposition:

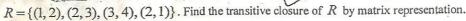


Find a non-empty set and a relation on the set that satisfy each of the following combinations of properties. Simultaneously, draw a digraph of each relation:

i) Reflexive and transitive but not symmetric.

 $(p \land q) \lor (p \lor r)$

- ii) Reflexive and transitive but not antisymmetric.
- Define Transitive Closure. Let $A = \{1, 2, 3, 4\}$ and
- [6]





Using principle of mathematical induction, show that

$$a+ar+ar^2+....+ar^{n-1}=\frac{a(1-r^n)}{1-r}$$
, $r\neq 0$.

b) Let $N = n^2 + n + 41$. Show that there are some values of n for which

N is a prime number, and others for which it is not. It follows that there is no inductive step which would show that $n^2 + n + 41$ is a prime number for all possible n. [6]

- A a) Let $f:A \to B$ and $g:B \to C$ be two functions. Show that
 - i) if both f and g are injective, then $g \circ f$ is injective
 - ii) if both f and g are surjective, then $g \circ f$ is surjective.
 - b) If $f: A \to B$ and $g: B \to C$ and $h: C \to D$, then show that

$$h \circ (g \circ f) = (h \circ g) \circ f$$

[6]

[12]

[6]

[4]

[8]

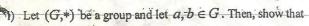
Write short notes of the following:

- (i) Labelled Trees (ii) Tree Searching (iii) Spanning Trees
- 6. a) If $(G_1, *_1)$ and $(G_2, *_2)$ are groups, then show that $G = G_1 \times G_2$ i.e., (G, *) is a group with binary

operation '*' defined by

operation ** defined by
$$(a_1, b_1) * (a_2, b_2) = (a_1 *_1 a_2, b_1 *_2 b_2)$$
 [8]

b) Consider the set Q of rational numbers, and let '*' be the operation on Q defined by a*b=a+b-ab. Show that (Q, *) is a semigroup. [4]



- (i) $(a^{-1})^{-1} = a$ (ii) $(a*b)^{-1} = b^{-1} * a^{-1}$

[6]

ii) If (G,*) is a group and if $a,b \in G$, then prove that

[6]

- (i) the equation a*x=b has a unique solution $x=a^{-1}*b \in G$
- (ii) the equation y * a = b has a unique solution $y = b * a^{-1} \in G$.